VALIDATION OF

STATEL RESULTS

NIST (National Institute of Standards and Technology) and its laboratory ITL (Information Technology Labolatory), located in USA, designed reference datasets with certified computational results that enable the objective evaluation of statistical software.

In order to certify users of StatEL about accuracy of its calculations, we confronted its results to certified results of ITL.

You can visit website “Statistical Reference Datasets“ of NIST on :
http://www.itl.nist.gov/div898/strd/

We used only datasets that are applicable with available tests of StatEL software, that is :

- Univariate statistics :
  - **PiDigits** (Lower level of difficulty)
  - **Lottery** (Lower level of difficulty)
  - **NumAcc2** (Average level of difficulty)
  - **NumAcc3** (Average level of difficulty)
  - **NumAcc4** (Higher level of difficulty)

- One-Way Anova :
  - **SiRstv** (Lower level of difficulty)
  - **SmLs01** (Lower level of difficulty)
  - **AtmWtAg** Average level of difficulty)
  - **SmLs06** (Average level of difficulty)
  - **SmLs07** (Higher level of difficulty)
  - **SmLs08** (Higher level of difficulty)

- Simple and multiple linear regression
  - **Norris**
  - **Longley**

Following results are displayed through screenshots of NIST results and those supplied by StatEL on Excel spreadsheet.

- Set **PiDigits** (Lower level of difficulty)

Certified Results :

Certified Values

Dataset Name: PiDigits

Procedure: Univariate Summary Statistics
Certification Method & Definitions

Data: 1 Response Variable (y)
5000 Observations
Lower Level of Difficulty
Observed Data

Model: 3 Parameters (μ, σ, ρ_1)

$$y_i = \mu + \epsilon_i$$

Parameter	Certified Estimate
μ	4.53480000000000
σ	2.86733906028871
ρ_1	-0.00355099287237972

Results of StatEL :

Results	
Nb of values	5000
Minimum	0
Maximum	9
Range	9
Mean	4,535
Variation Co	0,632
Standard Er	0,0406
CI 95% (Con	4,455 < m < 4,614
Standard De	2,867
Variance	8,222
Median	5
25th Percent	2
75th Percent	7
Inter Percent	5
Skewness (F	0,122
Kurtosis (Fis	-1,453

- Set **Lottery** (Lower level of difficulty)

Certified Results :

Certified Values

Dataset Name: Lottery

Procedure: Univariate Summary Statistics
Certification Method & Definitions

Data: 1 Response Variable (y)
218 Observations
Lower Level of Difficulty
Observed Data

Model: 3 Parameters (μ, σ, p_1)

$$y_i = \mu + \epsilon_i$$

Parameter	Certified Estimate
μ	518.958715596330
σ	291.699727470969
p_1	-0.120948622967393

Results of StatEL :

Results	
Nb of values	218
Minimum	4
Maximum	999
Range	995
Mean	519,0
Variation Co	0,562
Standard Er	19,76
CI 95% (Con	480,2 < m < 557,7
Standard De	291,7
Variance	85089
Median	522,5
25th Percent	272,8
75th Percent	779,3
Inter Percent	506,5
Skewness (F	-0,0891
Kurtosis (Fis	-1,257

- Set **NumAcc2** (Average level of difficulty)

Certified Results :

Certified Values

Dataset Name: NumAcc2

Procedure: Univariate Summary Statistics
Certification Method & Definitions

Data: 1 Response Variable (y)
1001 Observations
Average Level of Difficulty
Generated Data

Model: 3 Parameters (μ, σ, ρ_1)

$$y_i = \mu + \epsilon_i$$

Parameter	Certified Estimate	
μ	1.2	(exact)
σ	0.1	(exact)
ρ_1	-0.999	(exact)

Results of StatEL :

Results	
Nb of values	1001
Minimum	1,1
Maximum	1,3
Range	0,2
Mean	1,200
Variation Co	0,0833
Standard Er	3,16E-03
CI 95% (Cont	1,194 < m < 1,206
Standard De	0,1
Variance	0,0100
Median	1,2
25th Percent	1,1
75th Percent	1,3
Inter Percent	0,2
Skewness (F	2,93E-13
Kurtosis (Fis	-2,242

- Set **NumAcc3** (Average level of difficulty)

Certified Results :

Certified Values

Dataset Name: NumAcc3

Procedure: Univariate Summary Statistics
Certification Method & Definitions

Data: 1 Response Variable (y)
1001 Observations
Average Level of Difficulty
Generated Data

Model: 3 Parameters (μ, σ, ρ_1)

$$y_i = \mu + \epsilon_i$$

Parameter	Certified Estimate	
μ	1000000.2	(exact)
σ	0.1	(exact)
ρ_1	-0.999	(exact)

Results of StatEL :

Results			
Nb of values	1001		
Minimum	10000000,1		
Maximum	10000000,3		
Range	0,200		
Mean	10000000,2		
Variation Co	1,00E-08		
Standard Er	3,16E-03		
CI 95% (Cont	10000000,19 < m < 10000000,21		
Standard De	0,100		
Variance	0,0100		
Median	10000000,2		
25th Percent	10000000,1		
75th Percent	10000000,3		
Inter Percent	0,200		
Skewness (F	-2,55E-06		
Kurtosis (Fis	-2,242		

- Set **NumAcc4** (Higher level of difficulty)

Certified Results :

Certified Values

Dataset Name: NumAcc4

Procedure: Univariate Summary Statistics
Certification Method & Definitions

Data: 1 Response Variable (y)
1001 Observations
Higher Level of Difficulty
Generated Data

Model: 3 Parameters (μ, σ, ρ_1)

$$y_i = \mu + \epsilon_i$$

Parameter	Certified Estimate	
μ	10000000.2	(exact)
σ	0.1	(exact)
ρ_1	-0.999	(exact)

Results of StatEL :

Results			
Nb of values	1001		
Minimum	10000000,1		
Maximum	10000000,3		
Range	0,200		
Mean	10000000,2		
Variation Co	1,00E-08		
Standard Er	3,16E-03		
CI 95% (Con	10000000,19 < m < 10000000,21		
Standard De	0,100		
Variance	0,0100		
Median	10000000,2		
25th Percent	10000000,1		
75th Percent	10000000,3		
Inter Percent	0,200		
Skewness (F	-2,55E-06		
Kurtosis (Fis	-2,242		

ONE-WAY ANOVA

- Set **SiRstv** (Lower level of difficulty)

Certified Results :

Certified Values

Dataset Name: SiRstv

Procedure: Analysis of Variance
Certification Method & Definitions

Data: 1 Factor
5 Treatments
5 Replicates/Cell
25 Observations
3 Constant Leading Digits
Lower Level of Difficulty
Observed Data

Model: 6 Parameters ($\mu, \tau_1, \cdots, \tau_5$)
$y_{ij} = \mu + \tau_i + \epsilon_{ij}$

Source of Variation	Certified Degrees of Freedom	Certified Sums of Squares	Certified Mean Squares	Certified F Statistic
Between Instrument	4	5.11462616000000E-02	1.27865654000000E-02	1.18046237440255E+00
Within Instrument	20	2.16636560000000E-01	1.08318280000000E-02	
Certified R-Squared			1.90999039051129E-01	
Certified Residual Standard Deviation			1.04076068334656E-01	

Results of StatEL :

Results of One-way Anova	
Between-gr	0,0128
Within-grou	0,0108
F :	1,18
F lim :	2,866
p <	0,35

- Set **SmLs01** (Lower level of difficulty)

Certified Results :

Certified Values

Dataset Name: SmLs01

Procedure: Analysis of Variance
Certification Method & Definitions

Data: 1 Factor
9 Treatments
21 Replicates/Cell
189 Observations
1 Constant Leading Digit
Lower Level of Difficulty
Generated Data

Model: 10 Parameters $(\mu, \tau_1, \cdots, \tau_9)$
$y_{ij} = \mu + \tau_i + \epsilon_{ij}$

Source of Variation	Certified Degrees of Freedom	Certified Sums of Squares	Certified Mean Squares	Certified F Statistic
Between Treatment	8	1.68000000000000E+00	2.10000000000000E-01	2.10000000000000E+01
Within Treatment	180	1.80000000000000E+00	1.00000000000000E-02	
	Certified R-Squared		4.82758620689655E-01	
	Certified Residual Standard Deviation		1.00000000000000E-01	

Results of StatEL :

Results of One-way Anova	
Between-gro	0,210
Within-group	0,01
F :	21
F lim :	1,99
p <	0,00001

- **AtmWtAg** (Average level of difficulty)

Certified Results :

Certified Values

Dataset Name: AtmWtAg

Procedure: Analysis of Variance
Certification Method & Definitions

Data: 1 Factor
2 Treatments
24 Replicates/Cell
48 Observations
7 Constant Leading Digits
Average Level of Difficulty
Observed Data

Model: 3 Parameters (μ, τ_1, τ_2)
$y_{ij} = \mu + \tau_i + \epsilon_{ij}$

Source of Variation	Certified Degrees of Freedom	Certified Sums of Squares	Certified Mean Squares	Certified F Statistic
Between Instrument	1	3.63834187500000E-09	3.63834187500000E-09	1.59467335677930E+01
Within Instrument	46	1.04951729166667E-08	2.28155932971014E-10	
	Certified R-Squared		2.57426544538321E-01	
	Certified Residual Standard Deviation		1.51048314446410E-05	

Results of StatEL :

Results of One-way Anova	
Between-gr	3,64E-09
Within-grou	2,28E-10
F :	15,95
F lim :	4,052
p <	0,00023

- Set **SmLs06** (Average level of difficulty)

Certified Results :

Certified Values

Dataset Name: SmLs06

Procedure: Analysis of Variance
Certification Method & Definitions

Data: 1 Factor
9 Treatments
2001 Replicates/Cell
18009 Observations
7 Constant Leading Digits
Average Level of Difficulty
Generated Data

Model: 10 Parameters ($\mu, \tau_1, \ldots, \tau_9$)
$y_{ij} = \mu + \tau_i + \epsilon_{ij}$

Source of Variation	Certified Degrees of Freedom	Certified Sums of Squares	Certified Mean Squares	Certified F Statistic
Between Treatment	8	1.60080000000000E+02	2.00100000000000E+01	2.00100000000000E+03
Within Treatment	18000	1.80000000000000E+02	1.00000000000000E-02	
	Certified R-Squared		4.70712773465067E-01	
	Certified Residual Standard Deviation		1.00000000000000E-01	

Results of StatEL :

Results of One-way Anova	
Between-gro	20,01
Within-grou	0,0100
F :	2001
F lim :	1,939
p <	0,00001

- Set **SmLs07** (Higher level of difficulty)

Certified Results :

Certified Values

Dataset Name: SmLs07

Procedure: Analysis of Variance
Certification Method & Definitions

Data: 1 Factor
9 Treatments
21 Replicates/Cell
189 Observations
13 Constant Leading Digits
Higher Level of Difficulty
Generated Data

Model: 10 Parameters ($\mu, \tau_1, \cdots, \tau_9$)
$y_{ij} = \mu + \tau_i + \epsilon_{ij}$

Source of Variation	Certified Degrees of Freedom	Certified Sums of Squares	Certified Mean Squares	Certified F Statistic
Between Treatment	8	1.68000000000000E+00	2.10000000000000E-01	2.10000000000000E+01
Within Treatment	180	1.80000000000000E+00	1.00000000000000E-02	
	Certified R-Squared		4.82758620689655E-01	
	Certified Residual Standard Deviation		1.00000000000000E-01	

Results of StatEL :

Results of One-way Anova	
Between-gr	0,209897513
Within-grou	0,010000547
F :	20,99
F lim :	1,99
p <	0,00001

Note :
There is a slight difference between both results on this dataset whose data are 13 digits numbers.

- Set **SmLs08** (Higher level of difficulty)

Certified Results :

Certified Values

Dataset Name: SmLs08

Procedure: Analysis of Variance
Certification Method & Definitions

Data: 1 Factor
9 Treatments
201 Replicates/Cell
1809 Observations
13 Constant Leading Digits
Higher Level of Difficulty
Generated Data

Model: 10 Parameters ($\mu, \tau_1, \cdots, \tau_9$)
$y_{ij} = \mu + \tau_i + \epsilon_{ij}$

Source of Variation	Certified Degrees of Freedom	Certified Sums of Squares	Certified Mean Squares	Certified F Statistic
Between Treatment	8	1.60800000000000E+01	2.01000000000000E+00	2.01000000000000E+02
Within Treatment	1800	1.80000000000000E+01	1.00000000000000E-02	
	Certified R-Squared		4.71830985915493E-01	
	Certified Residual Standard Deviation		1.00000000000000E-01	

Results of StatEL :

Results of One-way Anova	
Between-gro	2,009018674
Within-group	0,010000545
F :	200,9
F lim :	1,944
p <	0,00001

Note :
There is a slight difference between both results on this dataset whose data are 13 digits numbers.

- Set **SmLs09** (Higher level of difficulty)

Certified Results :

Certified Values

Dataset Name: SmLs09

Procedure: Analysis of Variance
Certification Method & Definitions

Data: 1 Factor
9 Treatments
2001 Replicates/Cell
18009 Observations
13 Constant Leading Digits
Higher Level of Difficulty
Generated Data

Model: 10 Parameters $(\mu, \tau_1, \cdots, \tau_9)$
$y_{ij} = \mu + \tau_i + \epsilon_{ij}$

Source of Variation	Certified Degrees of Freedom	Certified Sums of Squares	Certified Mean Squares	Certified F Statistic
Between Treatment	8	1.60080000000000E+02	2.00100000000000E+01	2.00100000000000E+03
Within Treatment	18000	1.80000000000000E+02	1.00000000000000E-02	
	Certified R-Squared		4.70712773465067E-01	
	Certified Residual Standard Deviation		1.00000000000000E-01	

Results of StatEL :

Results of One-way Anova	
Between-gr	20,02466590
Within-grou	0,010000545
F :	2002
F lim :	1,939
p <	0,00001

Note :
There is a slight difference between both results on this dataset whose data are 13 digits numbers.

- Set **Norris**

Certified Results :

Certified Values

Dataset Name: Norris

Procedure: Linear Least Squares Regression
Certification Method & Definitions

Data: 1 Response Variable (*y*)
1 Predictor Variable (*x*)
36 Observations
Lower Level of Difficulty
Observed Data

Model: $y = \beta_0 + \beta_1 x + \epsilon$

Certified Regression Statistics

Parameter	Estimate	Standard Deviation of Estimate
β_0	-0.262323073774029	0.232818234301152
β_1	1.00211681802045	0.429796848199937E-03

Residual Standard Deviation	0.884796396144373
R-Squared	0.999993745883712

Results of StatEL :

Conclusions for analysis of Simple Linear Regression :
Linear regression model : Y = aX + b
y = 1,002*x - 0,2623
R² (Coefficient of Determination) : 99,999375%
This coefficient expresses percentage of variability that is expl
R²aj (Adjusted Coefficient of Determin 99,999356%
This coefficient allows to compare several linear models that d

- Set **Longley**

Certified Results :

Certified Values

Dataset Name: Longley

Procedure: Linear Least Squares Regression
Certification Method & Definitions

Data: 1 Response Variable (y)
6 Predictor Variable (x_1, ..., x_6)
16 Observations
Higher Level of Difficulty
Observed Data

Model: $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6 + \epsilon$

Certified Regression Statistics

Parameter	Estimate	Standard Deviation of Estimate
β_0	-3482258.63459582	890420.383607373
β_1	15.0618722713733	84.9149257747669
β_2	-0.358191792925910E-01	0.334910077722432E-01
β_3	-2.02022980381683	0.488399681651699
β_4	-1.03322686717359	0.214274163161675
β_5	-0.511041056535807E-01	0.226073200069370
β_6	1829.15146461355	455.478499142212

Residual Standard Deviation	304.854073561965
R-Squared	0.995479004577296

Results of StatEL :

Model of linear multiple regression :

y = 15,06 * x1 - 0,03582 * x2 - 2,02 * x3 - 1,033 * x4 - 0,0511 * x5 + 1829 * x6 - 3482259

Coefficient of multiple correlation (expresses intensity of the linear relation between the

r = 0,99774746

Test of r versus 0 : p < 0,00001

Coefficient of multiple determination (expresses percentage of variability that is explain

R^2 = 99,55%

Coefficient of adjusted multiple determination (allows to compare several linear models

R^2aj = 99,25%

Tests on model parameters :

Hypotheses :

H0 = studied parameter is not significantly different from 0.

H1 = studied parameter is significantly different from 0.

Variable	*Parameter*	*Standard-De*	*t*	*p*	*Confidence Interval (95%)*
Constant	-3482259	890420	-3,911	0,0036	[-5496529 ; -1467988]
x1	15,06	84,91	0,1774	0,86	[-177 ; 207,2]
x2	-0,03582	0,03349	-1,07	0,31	[-0,1116 ; 0,03994]
x3	-2,02	0,4884	-4,136	0,0025	[-3,125 ; -0,9154]
x4	-1,033	0,2143	-4,822	0,00094	[-1,518 ; -0,5485]
x5	-0,0511	0,2261	-0,2261	0,83	[-0,5625 ; 0,4603]
x6	1829	455,5	4,016	0,003	[798,8 ; 2860]

CONCLUSIONS

Except for data with more than 13 digits numbers, where Anova calculations differ very slightly, results of StatEL software are identical to certified results supplied by the NIST, on tested datasets.

Difference is due to the truncation error, the cancellation error, and/or the accumulation error. Any typical numerical algorithm will introduce computational inaccuracies, and will produce results which differ slightly from these certified values.

Consequently, if you need very precise and exact results for dataset whose values have more than 13 digits numbers, we suggest not to use StatEL, except if you are in a position to transform your data (see below).

One solution to avoid inaccuracies in calculation with such values is to subtract the leading constant from all the observations in that dataset before analyzing it.

Ex : if your data look all like 1000000000000.1, you just need to subtract 1000000000000 in order to launch calculations with 0.1…